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Dynamic response of a cracked piezoelectric ceramic under arbitrary electro-mechanical impact

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Abstract

The dynamic response of a cracked piezoelectric ceramic under in-plane electric and anti-plane mechanical impact is investigated by the integral transform method. The electric and mechanical loads are assumed to be arbitrary functions of time. It is shown that the dynamic crack-tip stress and electric displacement fields still have a square-root singularity. Numerical computations for the dynamic stress intensity factor show that the electric load has a significant influence on the dynamic response of stress field. On the other hand, the dynamic response of the electric field is determined solely by the applied electric field. The electric field will promote or retard the propagation of the crack depending on the time elapsed since the application of the external electro-mechanical loads.

1. Introduction

The elasto-static and dynamic response of piezoelectric materials and their failure modes have attracted considerable attention from many researchers recently. The crack extension force in a piezoelectric material in anti-plane shear (mode III) was calculated by Pak (1990) who showed that the mode III stress intensity factor will always be negative in the absence of mechanical loading. This result was confirmed in the work of Sosa and Pak (1990) who used an eigenfunction analysis. Sosa (1992) showed that even in mode I the stress intensity factor could be negative for some ratios of electrical to mechanical loading. Shindo and Ozawa (1990) investigated the steady-state response of a cracked piezoelectric material under the action of incident plane, harmonic waves. A finite crack in an infinite piezoelectric material under anti-plane dynamic electromechanical impact was investigated with the well-established integral transform methodology (Chen and Yu, 1997). Axisymmetric vibrations of a piezocomposite hollow cylinder were studied by Paul and Nelson (1996). The dynamic representation formulas and fundamental solutions for piezoelectricity had been proposed earlier by Khutoryansky and Sosa (1995).

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Descalu and Maugin (1995) studied the elastodynamic fracture of piezoelectric materials in the quasi elasto-static approximation using Stroh's formalism. They showed, in particular, that if a mode III crack is subjected gradually to remote mechanical and electrical loadings up to the moment when it begins to grow which are then held constant, the crack growth is strongly influenced by the applied electric field in a certain range. Below this range the electric field had no influence on the mechanical driving force, whereas above it the crack would stop growing.

The dynamic response of a cracked dielectric medium under the action of harmonic waves in a uniform electric field was studied by Shindo and his colleagues (Shindo et al., 1996). In their most recent work, Narita and Shindo (1998) investigated the scattering of Love waves by a surface-breaking crack normal to the interface in a piezoelectric layer over an elastic half plane. This work is very valuable for the reliability design of piezoelectric devices. Li and Mataga (1996a, b) studied a semi-infinite crack propagating in a piezoelectric material with electrode and vacuum boundary conditions on the crack surface, respectively. In their work, the transient dynamic electro-mechanical loads were taken into consideration. A new phenomenon about the surface waves in piezoelectric materials was reported.

Instead of the harmonic waves as the external loads, as in the work of Shindo and his colleagues, transient dynamic loads of arbitrary time variation are considered in the present work. The dynamic stresses and electric displacement around the crack tip are obtained. When the loads have the form of the Heaviside step function, the present results will reduce to those of Chen and Yu (1997).

2. Description of the problem and fundamental solution

Consider an infinite piezoelectric body containing a finite crack subjected to mechanical and electrical impacts. Let the length of the crack be $2a$. A set of Cartesian coordinates (x, y, z) is attached at the center of the crack for reference purposes. The x-axis is directed along the line of the crack and y-axis along the direction of the perpendicular bisector of the crack. The poled piezoelectric ceramic with z as the poling axis occupies the region $(-\infty < x < \infty, -\infty < y < \infty)$. It is sufficiently thick in the z-direction to allow a state of anti-plane shear to exist. Because of the assumed symmetry in geometry and loading, it is sufficient to consider the problem for $0 < x < \infty, 0 \leqslant y \leqslant \infty$ only.

The piezoelectric boundary-value problem for anti-plane shear is considerably simplified if we consider only the out-of-plane displacement and the in-plane electric fields such that the constitutive equations can be written as

$$\tau_{zk} = c_{44} w_{,k} + e_{15} \phi_{,k} \tag{1}$$

$$D_k = e_{15} w_{,k} - \varepsilon_{11} \phi_{,k} \tag{2}$$

where τ_{zk}, D_k $(k = x, y)$ are the anti-plane shear stress and in-plane electric displacement, respectively, c_{44}, e_{15}, ε_{11} are the shear modulus, piezoelectric coefficient and dielectric parameter, respectively; w and ϕ are the mechanical displacement and electric potential. The dynamic anti-plane governing equations for piezoelectric materials are (Shindo and Ozawa, 1990)

$$c_{44} \nabla^2 w + e_{15} \nabla^2 \phi = \rho\, \partial^2 w / \partial t^2 \tag{3}$$

$$e_{15}\nabla^2 w - \varepsilon_{11}\nabla^2\phi = 0 \tag{4}$$

where $\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$ is the two-dimensional Laplace operator. Body force, other than inertia, and the free charge are ignored in the present work. It is worth mentioning that the field eqns (3) and (4), for anti-plane deformation can also be derived by considering the so-called Bleustein–Gulayev SH surface waves (Maugin, 1988, 1993).

Substituting (4) into (3), gives the equation of wave motion

$$\nabla^2 w = c_2^{-2}\,\partial^2 w/\partial t^2 \tag{5}$$

in which $c_2 = \sqrt{\mu/\rho}$; $\mu = c_{44} + e_{15}^2/\varepsilon_{11}$.

Equation (5) has the following solution in Laplace transform domain with respect to time (Chen and Yu, 1997)

$$w^*(x,y,p) = \frac{2}{\pi}\int_0^\infty [A_1(s,p)\exp(-\gamma y) + A_2(s,p)\exp(\gamma y)]\cos(sx)\,\mathrm{d}s \tag{6}$$

where

$$\gamma(s,p) = \sqrt{s^2 + c_2^{-2}p^2} \tag{7a}$$

$$w^*(x,y,p) = \int_0^\infty w(x,y,t)\exp(-pt)\,\mathrm{d}t \tag{7b}$$

in which 'p' is the Laplace transform parameter. An asterisk denotes the Laplace transform throughout the paper.

Inserting (6) into (4), we have

$$\phi^*(x,y,p) = \frac{e_{15}}{\varepsilon_{11}}w^*(x,y,p) + \psi^*(x,y,p) \tag{8}$$

where

$$\psi^*(x,y,p) = \frac{2}{\pi}\int_0^\infty [A_3(s,p)\exp(-sy) + A_4(s,p)\exp(sy)]\cos(sx)\,\mathrm{d}s \tag{9}$$

Substituting (8) into the Laplace transforms of (1) and (2), we have

$$\tau_{zk}^* = \mu w_{,k}^* + e_{15}\psi_{,k}^* \tag{10}$$

$$D_k^* = -\varepsilon_{11}\psi_{,k}^* \tag{11}$$

The boundary conditions of the present problem are:

$$\tau_{zy}(x,\infty,t) = D_y(x,\infty,t) = 0, \quad 0 < x < \infty \tag{12a}$$

$$\tau_{zy}(x,0,t) = -\tau_0 f(t) \quad 0 < x < a \tag{12b}$$

$$D_y(x,0,t) = -D_0 g(t) \quad 0 < x < a \tag{12c}$$

$$w(x,0,t) = \phi(x,0,t) = 0, \quad x > a \tag{12d}$$

The Laplace transform of (12) yields

$$\tau^*_{zy}(x,\infty,p) = D^*_y(x,\infty,p) = 0, \quad 0 < x < \infty \tag{13a}$$

$$\tau^*_{zy}(x,0,p) = -\tau_0 f^*(p), \quad 0 < x < a \tag{13b}$$

$$D^*_y(x,0,p) = -D_0 g^*(p), \quad 0 < x < a \tag{13c}$$

$$w^*(x,0,p) = \phi^*(x,0,p) = 0, \quad x > a \tag{13d}$$

Substitution of (6), (8) and (9) into (10) and (11), and of the resulting expressions into (13a), yields

$$A_2(s,p) = A_4(s,p) = 0 \tag{14}$$

The problem therefore reduces to the determination of the two unknown functions $A_1(s,p)$ and $A_3(s,p)$. For this, the method developed by Chen and Yu (1997) can be used.

From (6) and (9), we have

$$\frac{\partial w^*(x,0,p)}{\partial y} = \frac{2}{\pi}\int_0^\infty -\gamma A_1(s,p)\cos(sx)\,\mathrm{d}s \tag{15}$$

$$\frac{\partial \psi^*(x,0,p)}{\partial y} = \frac{2}{\pi}\int_0^\infty -sA_3(s,p)\cos(sx)\,\mathrm{d}s \tag{16}$$

Inserting (6) and (8) into (10) and (11), and the resulting expressions into (13b) and (13c), gives

$$\frac{\partial w^*(x,0,p)}{\partial y} = -\frac{1}{\mu}[\tau_0 f^*(p) + e_{15}D_0 g^*(p)/\varepsilon_{11}], \quad 0 < x < a \tag{17}$$

$$\frac{\partial \psi^*(x,0,p)}{\partial y} = -\frac{D_0}{\varepsilon_{11}}g^*(p), \quad 0 < x < a \tag{18}$$

From (15)–(18), and the consideration of the boundary condition (13d), we get two pairs of dual integral equations

$$\frac{2}{\pi}\int_0^\infty A_1(s,p)\cos(sx)\,\mathrm{d}s = 0, \quad x > a$$

$$\frac{2}{\pi}\int_0^\infty \gamma(s,p)A_1(s,p)\cos(sx)\,\mathrm{d}s = \frac{1}{\mu}[\tau_0 f^*(p) + e_{15}D_0 g^*(p)/\varepsilon_{11}], \quad 0 < x < a \tag{19}$$

and

$$\frac{2}{\pi}\int_0^\infty A_3(s,p)\cos(sx)\,\mathrm{d}s = 0, \quad x > a$$

$$\frac{2}{\pi}\int_0^\infty sA_3(s,p)\cos(sx)\,\mathrm{d}s = \frac{D_0}{\varepsilon_{11}}g^*(p), \quad 0 < x < a \tag{20}$$

With the aid of Copson–Sih's method (Chen and Sih, 1977; see also Wang and Karihaloo, 1994), the solution of equation (19) can be written as follows

$$A_1(s,p) = \frac{\pi a^2}{2\mu}[\tau_0 f^*(p) + e_{15}D_0 g^*(p)/\varepsilon_{11}]\int_0^1 \sqrt{\xi}\phi_3^*(\xi,p)J_0(sa\xi)\,\mathrm{d}\xi \tag{21}$$

where $J_0(\cdot)$ is the zero-order Bessel function of the first kind, while $\phi_3^*(\xi,p)$ is determined by the following Fredholm integral equation of the second kind

$$\phi_3^*(\xi,p) + \int_0^1 K_3(\xi,\eta,p)\phi_3^*(\eta,p)\,\mathrm{d}\eta = \sqrt{\xi} \tag{22}$$

The kernel of the integral equation is

$$K_3(\xi,\eta,p) = (\xi\eta)^{1/2}\int_0^\infty s[\gamma(s/a,p)-1]J_0(s\xi)J_0(s\eta)\,\mathrm{d}s \tag{23}$$

Similarly, the solution of eqn (20) can be written as

$$A_3(s,p) = \frac{\pi a D_0}{2\varepsilon_{11}ps}J_1(as)g^*(p) \tag{24}$$

Substituting the results (6) and (9) and the resulting expressions into (10) and (11), gives the anti-plane mechanical displacement, electric potential, stress and electric displacement fields in the domain of Laplace transform. The inverse Laplace transform eventually gives the relevant time-dependent results.

3. Crack tip fields and dynamic intensities of stress and electric displacement

Let the crack tip be the origin of the polar coordinate system

$$r\exp(i\theta) = x - a + iy \tag{25}$$

where $i = \sqrt{-1}$ (Fig. 1).

From the above results, the stress and electric displacement around the crack tip can be expressed as (Chen and Sih, 1977)

$$\tau_{zy} + i\tau_{zx} = \frac{k_3^\tau(p)}{\sqrt{2\pi r}}\exp(-i\theta/2) + O(1) \tag{26}$$

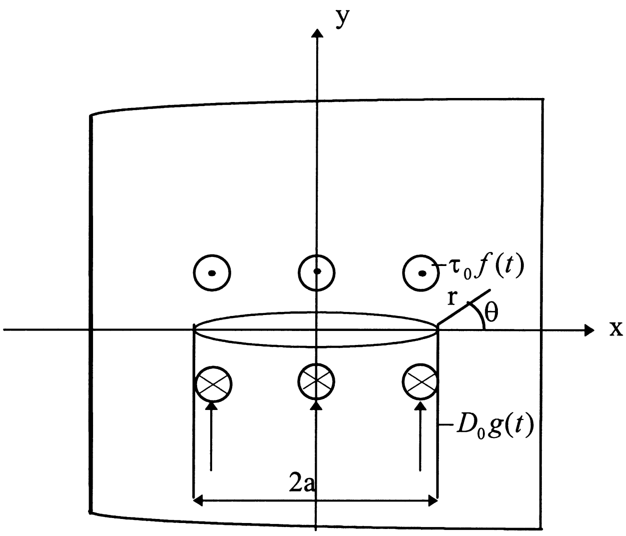

Fig. 1. Cracked piezoelectric ceramic under arbitrary anti-plane mechanical and electric impact.

$$D_y + iD_x = \frac{k_3^D(p)}{\sqrt{2\pi r}}\exp(-i\theta/2) + O(1) \tag{27}$$

where

$$k_3^\tau(t) = [\{\tau_0 M(t) + e_{15}D_0 N(t)/\varepsilon_{11}\} - e_{15}D_0 g(t)/\varepsilon_{11}]\sqrt{\pi a} \tag{28}$$

$$k_3^D(t) = D_0\sqrt{\pi a}g(t) \tag{29}$$

and

$$M(t) = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} \phi_3^*(1,p)f^*(p)\exp(pt)\,\mathrm{d}p \tag{30}$$

$$N(t) = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} \phi_3^*(1,p)g^*(p)\exp(pt)\,\mathrm{d}p \tag{31}$$

The function $\phi_3^*(1,p)$ can be calculated from (22). For a given form of the loading functions, the dynamic intensities of stress and electric displacement can be obtained from (28) and (29).

4. Numerical example and discussion

For simplicity, the functions $f(t)$ and $g(t)$ are assumed in the form of a Heaviside step function, that is $f(t) = g(t) = H(t)$, where, $H(t) = 1$ for $t >= 0$ and $H(t) = 0$ for $t < 0$. In the calculations to follow, the ratio $(e_{15}D_0/\varepsilon_{11})/\tau_0$ is assigned the value 0.0, 0.1, 0.2 or 0.5. The dynamic stress

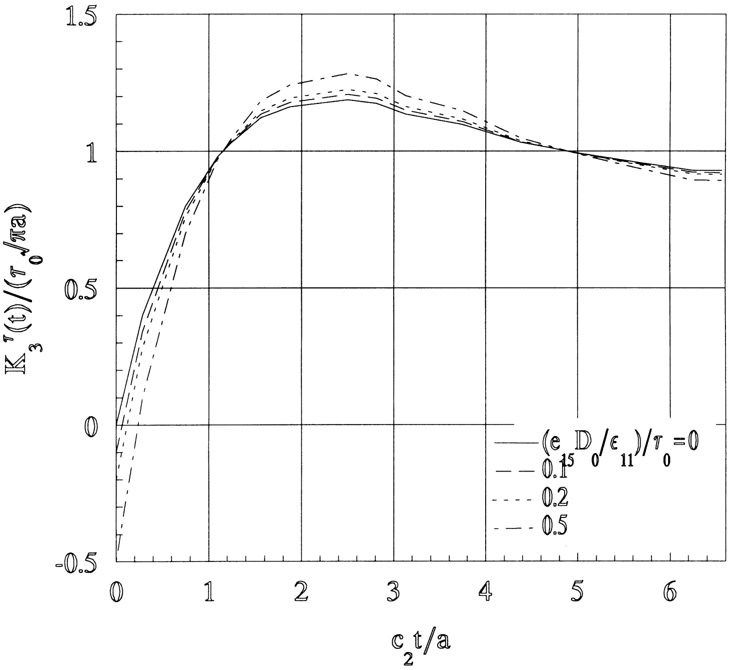

Fig. 2. Normalized dynamic stress intensity factor vs normalized time as a function of the ratio of the electric load to the mechanical load.

intensity factor (DSIF) is plotted in Fig. 2, which shows the influence of the applied electric field. At the very beginning of the loading process, the DSIF even becomes negative in the presence of the electric field. In other words, the electric field will retard the propagation of the crack. However, when the normalized time exceeds about 1.2, DSIF increases with increasing electric load, so that the electric field promotes the propagation of the crack. However, when the normalized time exceeds about 5.0, there is an opposite trend in the DSIF with the variation of electric field. It can be concluded that the dynamic electric field will promote or retard the propagation of the crack at different sages of the loading process. The effect of the electric field on the DSIF is independent of the direction of the electric field.

The situation considered here, namely that both the mechanical and electrical loadings are applied suddenly at $t = 0$, is different from that considered by Dascalu and Maugin (1995), namely that the mechanical and electrical loadings are increased gradually up to the moment of the onset of crack growth, whereafter they are held constant. For the elasto-dynamic situation considered by Dascalu and Maugin (1995), it was found that in the normalised time interval between 0 and 2, under an applied shear traction $\tau_0 = 4.2$ MPa, only an applied electric field E_2^∞ in the range 1×10^4–4.4×10^4 V/m has a strong influence on the crack growth. The crack evolution due to the applied shear traction is unaffected by the electric field if its strength is less than 1×10^4 V/m. On the other hand, the crack stops growing if this strength exceeds 4.4×10^4 V/m.

In the present notation, and using eqn (4.18) from the paper by Dascalu and Maugin (1995), the above range of E_2^∞ corresponds to the following range of the non-dimensional parameter

$0.6 \leqslant (e_{15}D_0)/(\varepsilon_{11}\tau_0) \leqslant 0.82$ for $\tau_0 = 4.2$ MPa and the material properties of PZT used in that paper. In other words, if $(e_{15}D_0)/(\varepsilon_{11}\tau_0) \leqslant 0.6$, the electric field has no influence on the stress intensity factor due to mechanical loading, if both the mechanical and electrical loadings are applied gradually. The present results show that the stress intensity factor is reduced even when this ratio is as small as 0.1. It can thus be concluded that electro-mechanical loading applied suddenly has a greater retardation influence upon the crack growth in the early stages (normalised time less than 1.2) than does the same loading, if it is applied gradually.

5. Conclusion

The anti-plane dynamic fracture problem of a cracked piezoelectric ceramic was reduced to the solution of two pairs of dual integral equations by using the methodology of integral transforms. By solving a Fredholm integral equation of the second kind, all the relevant quantities such as the anti-plane mechanical displacement, electric potential, stress and electric displacement, etc., can be easily obtained. The numerical example showed that the dynamic electric field will promote or retard the propagation of the crack at difference stages of the dynamic loading process. However, the dynamic electric displacement factor is coherent with the external electric load, independently of the mechanical load.

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